

# Investigation on the Tachyonic Neutrino

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## ABSTRACT

According to the experimental data, it is still controversial whether the neutrinos, especially the electron-neutrino and muon-neutrino, can be considered as the fermionic spinorial tachyons, and there is still no reliable report on the existence of the right-handed neutrinos. In this letter, we show that the neutrinos with the single handedness can not be the tachyons, but only those of the both handedness can be. Several implications of this result are discussed.

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According to recent experimental data [1], it is still controversial whether the neutrinos, especially the electron-neutrino and muon-neutrino, can be considered as the tachyons [2], which is the hypothetical objects moving faster than light in vacuum <sup>1</sup>, i.e., fermionic spinorial tachyons. Furthermore, there is still no reliable report on the existence of the right-handed neutrinos.

In this letter, we firstly present a proof that the single handedness particles can not be the tachyons, but only those of the both handedness can be.

Let us start by considering the tachyonic Dirac equation [3], which is Lorentz covariant and represents the tachyonic particle in 4×4 representation in the physical 3-space and 1-time dimension as follows

$$(i\gamma^\mu\partial_\mu - \lambda_T)\psi(x) = 0, \quad (1)$$

where  $\lambda_T$  is generally given by

$$\lambda_T = iaI + b\gamma^5 \quad (2)$$

with the real constants  $a$  and  $b$ , and  $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$  <sup>2</sup>. By way of the group theory, this finite-component (here four-component) theory, of course, involves a non-unitary representation of the Lorentz transformation. Here,  $\gamma$ -matrices satisfies the usual Clifford algebra  $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$  in our convention  $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ . In this case the mass squared  $m^2$  of the spinor  $\psi(x)$  is found to be

$$m^2 = (\gamma^0\lambda_T)^2 = -a^2 - b^2 < 0 \quad (3)$$

implying clearly the tachyonic movement, by comparing with the Klein-Gordon equation

$$(\partial^\mu\partial_\mu + m^2)\psi(x) = 0.$$

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<sup>1</sup> The pioneering works on the usual tachyon theory were given by Ref. [2]. Later development were not deviated much far from the lines of these papers. However, it has been pointed out recently that these formulations are incomplete even at the classical (non-quantum theoretical) level and the theory was reformulated by authors, On the Foundation of the Relativistic Dynamics with the Tachyon, Sogang Univ. Report No. SOGANG-HEP 197/95, hep-th/9506082. According to the our formulation, the rest mass of the tachyon is not anymore Lorentz scalar but the sign may be changed under the Lorentz transformation depending on it's velocity for consistency. But even in this formulation, the mass squared  $m^2$  is still Lorentz invariant. Hence, in our interesting wave equation in this letter, the linear wave equation, this unusual property of the mass may have a role for the covariance of the wave equation. But we will not quote here this new formulation because it is sufficient to use, to derive our result, only the fact that mass squared  $m^2$  for the tachyon is negative valued Lorentz scalar, which is the same for the both old and new formulations.

<sup>2</sup> The general wave equations for the spinorial bradyon (the object moving slower than light) and luxon (the object moving with the velocity of light) can be described similarly by  $\lambda_B = cI + id\gamma^5$  and  $\lambda_L = f_\pm(I \pm \gamma^5)$  for real numbers  $c$ ,  $d$  and complex number  $f_\pm$ , respectively. Furthermore, note that we can use the chiral transformation to transform the pseudoscalar or scalar part away. But, in that case the physics described by the spinor is changed due to the non-invariance of the theories under the transformation.

Moreover, in this wave equation the usual vector current  $J^\mu \equiv \bar{\psi}\gamma^\mu\psi$  is not conserved anymore for any non-zero  $a$  and  $b$ , but the axial current  $J_5^\mu \equiv \bar{\psi}\gamma^\mu\gamma^5\psi$  can be conserved for the case of  $a = 0$  discarding the physically uninteresting case of  $b = \infty$ , which corresponds to the infinitely massive immovable particle because of

$$\partial_\mu J^\mu = 2[a\bar{\psi}\psi - ib\bar{\psi}\gamma^5\psi], \quad (4)$$

$$\partial_\mu J_5^\mu = \frac{-ia}{b}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}_\mu \psi, \quad (5)$$

where  $\overleftrightarrow{\partial}_\mu$  acts as  $F \overleftrightarrow{\partial}_\mu G = F\partial_\mu G - (\partial_\mu F)G$  for some function  $F$  and  $G$ . The corresponding Lagrangian density, which is Hermitian or anti-Hermitian depending on the statistics <sup>3</sup> is uniquely found to be (up to normalization constant) <sup>4</sup>

$$\mathcal{L}_T = -\frac{i}{2}\bar{\psi}\gamma^5\gamma^\mu \overleftrightarrow{\partial}_\mu \psi + \bar{\psi}\gamma^5\lambda_T\psi. \quad (6)$$

Furthermore, the corresponding canonical Hamiltonian becomes

$$\begin{aligned} H_T &= \int d^3x \left( \Pi_\psi \dot{\psi} + \dot{\psi}^\dagger \Pi_{\psi^*} - \mathcal{L}_T \right) \\ &= \int d^3x \psi^\dagger \gamma^5 h_T \psi, \end{aligned} \quad (7)$$

which is Hermitian or anti-Hermitian depending on the Hermiticity of the Lagrangian. Here, the canonical momenta are  $\Pi_\psi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}}|_r = -(i/2)\bar{\psi}\gamma^5\gamma^0$  and  $\Pi_{\psi^*} \equiv \frac{\partial \mathcal{L}}{\partial \dot{\psi}^*}|_l = -(i/2)\gamma^5\psi$ , where the subscripts  $r$  and  $l$  represent the right and left derivatives, respectively, <sup>5</sup> and  $h_T$  is the one-particle Hamiltonian

$$h_T = \vec{\alpha} \cdot \vec{p} + ia\gamma^0 + b\gamma^0\gamma^5, \quad (8)$$

with  $\vec{\alpha} = \gamma^0\vec{\gamma}$ . Note that for the case of the anti-Hermitian Hamiltonian the normalization constant of the Lagrangian density  $\mathcal{L}_T$  should be adjusted such that the Hamiltonian is Hermitian. But, in this letter we will preserve the normalization as the Lagrangian (6) since this is not important problem in our analysis.

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<sup>3</sup> In general, this Hermiticity or anti-Hermiticity guarantee the consistency of the Euler-Lagrange equations derived from the variation of  $\psi$  and  $\psi^*$ . But, the Hermiticity is favored such that in this case the Hermiticity of the Hamiltonian is also guaranteed.

<sup>4</sup> Including the bradyon and luxon cases, all cases are described unifiedly by  $\mathcal{L}_\xi = (i/2)\bar{\psi}\xi\gamma^\mu \overleftrightarrow{\partial}_\mu \psi - \bar{\psi}\xi\lambda\psi$  with  $\xi = -\gamma^5$ ,  $I$ , and  $(I \pm \gamma^5)$  for the cases of tachyon, bradyon, and luxon, respectively.

<sup>5</sup> The use of the different derivatives for  $\Pi_\psi$  and  $\Pi_{\psi^*}$  together with the unusual definition of the Hamiltonian (7) (the formal sign of the second term is different with the usual one) are devised in order that these can be defined without explicit consideration of the exchange algebras.

Now, in order to treat the handedness problem, let us explicitly consider the chiral representation, i.e.,

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad \gamma^\mu = \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (9)$$

where  $\psi_L$  and  $\psi_R$  are the two-component spinors that transform as  $(\frac{1}{2}, \mathbf{0})$  and  $(\mathbf{0}, \frac{1}{2})$  representations of the Lorentz group, respectively, and  $\sigma^\mu \equiv (1, \vec{\sigma})$ ,  $\bar{\sigma}^\mu \equiv (1, -\vec{\sigma})$ . In this representation, Eq. (1) reduces to two sets of equations

$$i\bar{\sigma}^\mu \partial_\mu \psi_R - (ia + b)\psi_L = 0, \quad (10)$$

$$i\sigma^\mu \partial_\mu \psi_L - (ia - b)\psi_R = 0, \quad (11)$$

where both  $\psi_L$  and  $\psi_R$  are the tachyonic spinors having the same mass squared  $m^2 = -a^2 - b^2$  as that of  $\psi$ . Then, the corresponding Lagrangian density and Hamiltonian become

$$\mathcal{L}_T = \frac{i}{2} \psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L - \frac{i}{2} \psi_R^\dagger \bar{\sigma}^\mu \overleftrightarrow{\partial}_\mu \psi_R + (ia + b) \psi_L^\dagger \psi_R + (-ia + b) \psi_R^\dagger \psi_L, \quad (12)$$

$$H_T = \int d^3x \left[ -\frac{i}{2} \psi_L^\dagger \sigma^i \overleftrightarrow{\partial}_i \psi_L - \frac{i}{2} \psi_R^\dagger \bar{\sigma}^i \overleftrightarrow{\partial}_i \psi_R - (ia + b) \psi_L^\dagger \psi_R - (-ia + b) \psi_R^\dagger \psi_L \right]. \quad (13)$$

Moreover, the axial current density  $J_5^\mu$ , which is conserved for the case of  $a = 0$ , becomes

$$J_5^\mu = \psi_L^\dagger \sigma^\mu \psi_L - \psi_R^\dagger \bar{\sigma}^\mu \psi_R \quad (14)$$

explicitly showing the non positive-definiteness (more exactly the sign indefiniteness) of  $J_5^0 = \psi_L^\dagger \psi_L - \psi_R^\dagger \psi_R$  such that the usual probability interpretation is questionable in this case. However, according to our usual experiences in the second quantization theory, this problem is not so serious one. In this case it is well interpreted only if we can develop a theory with (lower) bounded Hamiltonian irrespective on the non-existence of the positive definite conserved current density, of course, together with other fundamental principles like as the microscopic causality and the Lorentz covariance. The bradyonic scalar, spinor, and vector particles are the examples [4]. But, unfortunately this scenario for the tachyons can not be checked at present because there are no known consistent second quantization rules for the spinorial tachyon. Actually even for the tachyonic scalar, which will be the most simple case in the tachyonic particles, the consistent quantization rule has not been known so far [5]. However, we will show that especially for single handedness spinorial particles we have a stringent situation for the existence of the tachyons, i.e., the theory of the tachyons with the single handedness like as the Majorana particles are not consistent even at the level of the first quantization without knowing the full situation of the second quantized theory.

To prove this, let us consider the single handedness tachyons, which is obtained directly from the both handedness theory by reducing the handedness. On the other hand, we note that the most general covariant reduction of the handedness should be obtained, if the single-handed theory as well as the both-handed theory are existed, by

$$\psi_R = -\alpha\sigma^2\psi_L^* \quad (15)$$

or

$$\psi_L = \beta\sigma^2\psi_R^* \quad (16)$$

since this is the most general relation connecting the two different handedness spinors  $\psi_R$  and  $\psi_L$  within the transformation theory of the spinor [6], and hence the four-component spinor  $\psi$  in Eq. (9) becomes

$$\psi = \begin{pmatrix} \psi_L \\ -\alpha\sigma^2\psi_L^* \end{pmatrix} \quad (17)$$

or

$$\psi = \begin{pmatrix} \beta\sigma^2\psi_R^* \\ -\psi_R \end{pmatrix} \quad (18)$$

for the left-handedness or the right-handedness only theories with the constant  $\alpha$  and  $\beta$ , respectively. But, it is important to note that the *handedness – reduction*, by it's means, should not change the physical contents of each handedness spinor of the original theory except reducing the handedness.

Now, for the application to the neutrinos we consider the only left-handedness case (17). But, the conclusion is also the same for the only right-handedness case (18). We first consider the reduction in the wave equations, i.e., the reduction from the wave equations (10) and (11). By putting spinor relation (15) into Eqs. (10) and (11), or equivalently (17) into Eq. (1) we obtain

$$i\alpha\bar{\sigma}^\mu\sigma^2\partial_\mu\psi_L^* - (ia + b)\psi_L = 0, \quad (19)$$

$$i\sigma^\mu\partial_\mu\psi_L + \alpha(ia - b)\sigma^2\psi_L^* = 0. \quad (20)$$

By inspection, it is easy to expect that Eqs. (19) and (20) would be the complex conjugations of each other if these equations are consistent. However, surprisingly this is not the case. To see this, we apply the complex conjugation to Eq. (19), and use the identity of the Pauli's spin matrices

$$\bar{\sigma}^\mu\sigma^2 = \sigma^2\widetilde{\sigma}^\mu$$

with the transposed matrices  $\widetilde{\sigma}^\mu$ . Then we find that Eq. (19) can be written as follows

$$i\sigma^\mu\partial_\mu\psi_L - \frac{1}{\alpha^*}(ia - b)\sigma^2\psi_L^* = 0, \quad (21)$$

which should be equal to Eq. (20) for consistency. But, this equation is equal to Eq. (20) only if

$$\alpha\alpha^* = -1 \quad (22)$$

is satisfied since  $a$  or  $b$  is non-zero for the tachyons and  $\sigma^\mu\partial_\mu\psi_L \neq 0$  in general. But, note that this has no solution within the complex number <sup>6</sup>. Hence, the single handedness spinorial tachyon wave equation, which is reduced from the both handedness spinorial tachyon wave equation, is inconsistent for any non-zero mass and any complex-number  $\alpha$ . Furthermore, we note that this inconsistency can not be attributed to the matter of the handedness reduction method due to its general form, and hence it should be attributed to the matter of the spinorial tachyon wave equation itself. In other words, although we have shown that the inconsistency of only the reduced single-handed tachyons from the both-handed tachyons, this inconsistency is actually a genuine property of the single-handed tachyons themselves irrespective of the handedness reduction method. Of course, for the zero-mass case of  $a = b = 0$  the consistency is trivially satisfied as is well-known, for example, in the Weyl equation. Furthermore, note that this result is derived without restricting to any statistics. Usually the statistics of the particles is not determined by their wave equation, but by the consistency of the second quantized theory with many fundamental principles like as the ones mentioned previously. However, in many cases <sup>7</sup> the statistical nature of the fields can be also predicted from the classical Lagrangians or Hamiltonians.

We now examine how the inconsistency at the level of equations of motion is transferred to the Lagrangian and Hamiltonian by explicitly considering the exchange algebras of the fields, which will be related to the statistics of the fields after the second quantization. Moreover, since all the results from the Hamiltonian analysis can be also obtained in the Lagrangian analysis, we only consider here the latter analysis to avoid the duplication. To this end, let us replace  $\psi_R$  with  $\psi_L$  by Eq. (15) in the Lagrangian (12). Then, the Lagrangian reduces to

$$\mathcal{L}_T = \frac{i}{2}\psi_L^\dagger\sigma^\mu\overleftrightarrow{\partial}_\mu\psi_L - \frac{i}{2}|\alpha|^2\widetilde{\psi}_L\widetilde{\sigma}^\mu\overleftrightarrow{\partial}_\mu\psi_L^* - \alpha(ia + b)\psi_L^\dagger\sigma^2\psi_L^* - \alpha^*(-ia + b)\widetilde{\psi}_L\sigma^2\psi_L \quad (23)$$

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<sup>6</sup> We may introduce an hypothetical number  $\alpha$  having the property  $\alpha\alpha^* < 0$  by enlarging the set of numbers in mathematics together with the spinor having the property of the hypothetical number for each component. But, in this letter we only confine ourselves to the usual number theory for simplicity. If this possibility is considered, our conclusion will be drastically changed. See Footnote 10 for this problem.

<sup>7</sup> The real scalar field, complex scalar field when decomposed into two real scalar fields, massless and massive vector particles are the cases. In these cases the classical Lagrangian and Hamiltonian become vanishing for wrong statistics.

without assuming the exchange algebras of the spinors. But, since the complete analysis of the Lagrangian is possible only after explicit consideration of the exchange algebras of the fields, we consider here the two typical cases, i.e., anti-commuting and commuting fields, which will be corresponded to the fermion and boson statistics after the second quantization, respectively. More general exchange algebras might be introduced, but that is not essential for our consideration.

Firstly, let us consider the case of the anti-commuting fields, i.e.,

$$\begin{aligned}\psi_L^\dagger(x)\psi_L(y) &= -\psi_L(y)\psi_L^\dagger(x), \\ \psi_L(x)\psi_L(y) &= -\psi_L(y)\psi_L(x).\end{aligned}\tag{24}$$

After the second quantization these relations would be centrally deformed with the operator-valued  $\psi_L$  and  $\psi_L^\dagger$ . Now, with the algebra (24), the Lagrangian (23) reduces to

$$\mathcal{L}_T = -\frac{i}{2}(|\alpha|^2 - 1)\psi_L^\dagger\sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L - \alpha(ia + b)\psi_L^\dagger\sigma^2\psi_L^* - \alpha^*(-ia + b)\widetilde{\psi}_L\sigma^2\psi_L.\tag{25}$$

For the case of  $|\alpha| = 1$ , which is the only case of the consistent reduction of the bradyon, which will be shown later, the Lagrangian has no kinetic terms, i.e.,

$$\mathcal{L}_T = -\alpha(ia + b)\psi_L^\dagger\sigma^2\psi_L^* - \alpha^*(-ia + b)\widetilde{\psi}_L\sigma^2\psi_L.\tag{26}$$

Hence the handedness reduction of the relation (15) is failed in this case because the handedness reduction changes the physical content of the spinor  $\psi_L$ , i.e., it's mass can be considered to become infinitely large upon the reduction even when the mass of the original both-handed Lagrangian (12) is finite. However, note that this result is consistent with the equation of motion analysis. In other words, for the case of  $|\alpha| = 1$  Eq. (21) equivalent to Eq. (19), becomes

$$i\sigma^\mu\partial_\mu\psi_L - \alpha(ia - b)\sigma^2\psi_L^* = 0\tag{27}$$

such that this equation is consistent with another equation (20) only when

$$i\sigma^\mu\partial_\mu\psi_L = 0,\tag{28}$$

or

$$(ia - b)\sigma^2\psi_L^* = 0\tag{29}$$

is satisfied. The first case (28) corresponds to our case of the anti-commuting spinor. The second one (29) corresponds to the commuting spinor case, which will be shown shortly. Furthermore, for the case of  $|\alpha| \neq 1$  the Lagrangian becomes

$$\mathcal{L}_T = -(|\alpha|^2 - 1)\left[\frac{i}{2}\psi_L^\dagger\sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L + \frac{f}{2}\psi_L^\dagger\sigma^2\psi_L^* + \frac{f^*}{2}\widetilde{\psi}_L\sigma^2\psi_L\right]\tag{30}$$

with

$$f = \frac{2\alpha(ia + b)}{|\alpha|^2 - 1}, \quad (31)$$

but the spinor  $\psi_L$ , now, has the mass squared  $m^2$  as follows

$$m^2 = |f|^2 = \frac{4|\alpha|^2(a^2 + b^2)}{(|\alpha|^2 - 1)^2}, \quad (32)$$

which is not the same as Eq. (3), that of in the original Lagrangian (12) for any  $\alpha$  with  $|\alpha| \neq 1$  such that in this case the handedness reduction is also failed. Hence, we can conclude that there are no consistent handedness reductions for any  $\alpha$  for the case of the anti-commuting fields reproducing the result, which is drawn from the equations of motion in this case.

Secondly, let us consider the case of the commuting fields <sup>8</sup>, i.e.,

$$\begin{aligned} \psi_L^\dagger(x)\psi_L(y) &= \psi_L(y)\psi_L^\dagger(x), \\ \psi_L(x)\psi_L(y) &= \psi_L(y)\psi_L(x). \end{aligned} \quad (33)$$

Then, the Lagrangian (23) reduces

$$\mathcal{L}_T = \frac{i}{2}(|\alpha|^2 + 1)\psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L. \quad (34)$$

But, the spinor  $\psi_L$  in this action has different mass from that of the original both-handed Lagrangian (12), i.e., zero-mass for any  $\alpha$ . Hence, the handedness reduction is not also consistent for any  $\alpha$  in this case. This reproduces the result drawn from the equations of motion for the case of the commuting fields. In this way we have shown that the single handedness spinorial tachyon Lagrangian can not be isolated from the both handedness spinorial tachyon Lagrangian for the both anti-commuting and commuting fields, which directly implying the non-existence of the single handedness spinorial tachyon Lagrangian due to the same reasoning in the analysis of the equations of motion.

Although our main concern is about the tachyons, it will be instructive to compare our results with the well-known case of the bradyonic particles allowing the single-handed particles. To this end, we first note that the wave equation for the general wave equation for the bradyon [3, 6]<sup>2,4</sup> can be written as

$$(i\gamma^\mu \partial_\mu - \lambda_B) \psi(x) = 0, \quad (35)$$

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<sup>8</sup> Since the usual spin-statistics connections may not be applied to the tachyon case, this would-be wrong statistics may not be ruled out from the start although we can not obtain an affirmative answer for the statistics of the tachyon in our problem.



where  $\lambda_B = (cI + id\gamma^5)$ , and which has the mass squared  $m^2$  for the spinor  $\psi$  as

$$m^2 = (\gamma_0 \lambda_B)^2 = c^2 + d^2 > 0 \quad (36)$$

implying the bradyonic movement. Hence, the wave equation for the chiral representation is written as

$$i\bar{\sigma}^\mu \partial_\mu \psi_R - (c + id)\psi_L = 0, \quad (37)$$

$$i\sigma^\mu \partial_\mu \psi_L - (c - id)\psi_R = 0, \quad (38)$$

where both  $\psi_L$  and  $\psi_R$  have the same mass as  $\psi$  in Eq. (36).

Now, if we try the handedness reduction by Eq. (15) as in the case of the tachyon, then Eqs. (37) and (38) become

$$-i\alpha\bar{\sigma}^\mu \sigma^2 \partial_\mu \psi_L^* - (c + id)\psi_L = 0, \quad (39)$$

$$i\sigma^\mu \partial_\mu \psi_L + \alpha(c - id)\sigma^2 \psi_L^* = 0. \quad (40)$$

These two equations would be the complex conjugations of each other if the reduction is consistent. To investigate this, we apply the complex conjugation to Eq. (39), and use the Pauli's matrices identity as in the tachyon case. Then, we obtain

$$i\sigma^\mu \partial_\mu \psi_L + \frac{(c - id)}{\alpha^*} \sigma^2 \psi_L^* = 0, \quad (41)$$

which should be the same as Eq. (40) for consistency. But, this equation is equal to Eq. (40), for non-zero  $c$  or  $d$  in order not to discuss the trivially satisfying case of the luxons, only if

$$\alpha\alpha^* = 1 \quad (42)$$

is satisfied<sup>9</sup>. This result is derived without restricting to any exchange algebras of the fields.

Now, let us examine how this condition is derived from the Lagrangian analysis. To this end, we note that the Lagrangian corresponding to the equation of motion (35) is

$$\mathcal{L}_B = \frac{i}{2} \bar{\psi} \gamma^\mu \overleftrightarrow{\partial}_\mu \psi - \bar{\psi} (c + id\gamma^5) \psi, \quad (43)$$

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<sup>9</sup> For the case  $\alpha = 1$ , i.e.,  $\psi_R = -\sigma^2 \psi_L^*$ , the four-component spinor  $\psi$  is usually named as the Majorana spinor, and in this case the spinor  $\psi$  of the representation (9) is self charge-conjugate, i.e.,  $\psi_c = \psi$  under the usual charge conjugation

$$\psi_c = \begin{pmatrix} \sigma^2 \psi_R^* \\ -\sigma^2 \psi_L^* \end{pmatrix}$$

implying (electric) charge neutral. The other cases of (42) are also charge neutral although the spinors are not self charge-conjugate. Hence, for the case of the bradyons, any reduced spinors (17) or (18) are charge neutral.

and hence the chiral form is

$$\mathcal{L}_B = \frac{i}{2}\psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L + \frac{i}{2}\psi_R^\dagger \overline{\sigma}^\mu \overleftrightarrow{\partial}_\mu \psi_R + (-c + id)\psi_L^\dagger \psi_R + (-c - id)\psi_R^\dagger \psi_L. \quad (44)$$

Now, in order to reduce this both handedness Lagrangian to single handed (here left-handed) Lagrangian, let us replace  $\psi_R$  with  $\psi_L$  by Eq. (15). Then, the Lagrangian (44) reduces to

$$\mathcal{L}_B = \frac{i}{2}\psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L + \frac{i}{2}|\alpha|^2 \widetilde{\psi}_L \overline{\sigma}^\mu \overleftrightarrow{\partial}_\mu \psi_L^* - \alpha(-c + id)\psi_L^\dagger \sigma^2 \psi_L^* - \alpha^*(-c - id)\widetilde{\psi}_L \sigma^2 \psi_L \quad (45)$$

without restricting any exchange algebras of the fields. As in parallel with the case of the tachyons let us consider explicitly the anti-commuting and commuting fields. So we first consider the case of the anti-commuting fields such that the algebra (24) is satisfied. In this case, the Lagrangian (45) reduces to

$$\mathcal{L}_B = (|\alpha|^2 + 1) \left[ \frac{i}{2}\psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L + \frac{g}{2}\psi_L^\dagger \sigma^2 \psi_L^* + \frac{g^*}{2}\widetilde{\psi}_L \sigma^2 \psi_L \right] \quad (46)$$

with

$$g = \frac{2\alpha(-c + id)}{|\alpha|^2 + 1},$$

but the spinor  $\psi_L$  has the mass squared  $m^2$  as

$$m^2 = |g|^2 = \frac{4|\alpha|^2(c^2 + d^2)}{(|\alpha|^2 + 1)^2}, \quad (47)$$

which is not the same as Eq. (36), that of in the original both-handed Lagrangian (45) unless  $|\alpha| = 1$ . Hence, the handedness reduction method, which by definition should not change any physical properties but pick up only one-handed part, is consistent only for  $|\alpha| = 1$ , which reproduces the condition (42) for the anti-commuting fields.

Secondly, if we consider the case of the commuting fields such that algebra (33) is satisfied, the Lagrangian (45) reduces to

$$\mathcal{L}_B = \frac{i}{2}(1 - |\alpha|^2)\psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L. \quad (48)$$

But, the spinor  $\psi_L$  in this action has different mass, i.e., zero-mass as that of the original Lagrangian (45) unless  $|\alpha| = 1$  such that the handedness reduction is not consistent unless  $|\alpha| = 1$  due to the same reason as in the previous fermionic case. This is a reproduction of the condition (42) for the commuting fields. However, for the case of  $|\alpha| = 1$ , the Lagrangian becomes vanishing such that the commuting spinor may be excluded in this sense. This can be considered as the spin-statistics connection at the classical level. So we have found that the

handedness problem of the spinorial bradyons is drastically different from that of the tachyons: the single-handed particles are allowed for the bradyons but not for the tachyons.

Now, finally we will show that these results are consistent with those of the theory without considering the handedness reduction, i.e., by considering the single handedness two-component theory from the start. To this end, we first note that the most general form of the single-handed (here left-handed) and Lorentz-invariant two-component spinor Lagrangian can be expressed as [6]

$$\mathcal{L} = \frac{i}{2} \psi_L^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \psi_L + \frac{h}{2} \psi_L^\dagger \sigma^2 \psi_L^* + \frac{h^*}{2} \widetilde{\psi}_L \sigma^2 \psi_L, \quad (49)$$

which is exactly the same as the Lagrangian (30), the left handedness Lagrangian reduced from the both-handed original Lagrangian (23) by (15) or from (6) by (17) except the normalization factor and produces the wave equations

$$i\sigma^\mu \partial_\mu \psi_L + h\sigma^2 \psi_L^* = 0, \quad (50)$$

and

$$i\overline{\sigma}^\mu \sigma^2 \partial_\mu \psi_L^* + h^* \sigma^2 \psi_L = 0 \quad (51)$$

by varying the Lagrangian (49) with respects to  $\psi_L^*$  and  $\psi_L$  respectively. Note that these two equations are consistent as the complex conjugated one with each others. Now, if we calculate the mass of the spinor  $\psi_L$  by comparing the Klein-Gordon equation, we find that as Eq. (32)

$$m^2 = |h|^2 \geq 0 \quad (52)$$

such that it is easily concluded that the single-handed spinorial tachyon is impossible although the corresponding bradyon is possible one in general.

In conclusion, we have shown in this letter that the single-handed spinorial tachyons can not exist for any exchange algebras of the spinor fields in three different approaches, i.e., a) by proving the non-existence of the reduction of the single-handed two-component tachyon from the both-handed spinorial four-component tachyon without restricting any exchange algebras of the fields, b) by proving the non-existence of the previous reduction in the Lagrangian with the specific exchange algebras of the spinors, i.e., anti-commuting and commuting spinors which will be corresponded to the fermion and boson statistics after the second quantization, respectively, and c) by directly proving the non-existence of the tachyonic mode for the most general form of the single-handed two-component spinorial spinor Lagrangian. Hence, we conclude that at least the both handedness are required in order that the spinorial tachyon may exist.

Now, let us consider the applicability of our result to the neutrinos whose tachyonic property is still controversial. Since it is strongly believed that, if a neutrino has the tachyonic property,

this should be described by the Dirac-like equation (1) in order to have the consistency with the traditional Dirac-equation treatment of the massless neutrinos, the tachyonicity of the neutrinos is governed by our result. However, there is still no reliable report on the existence of the right-handed neutrinos. Hence, in this situation as far as there are no right-handed neutrinos, we conclude that our observing left-handed neutrinos can not be the tachyons. In this way the proposal of the tachyonic neutrino is ruled out with the same accuracy of the non-existence of the right-handed neutrino. However, if we really can confirm the tachyonicity of several neutrinos in the future, this will give an affirmative clue to the existence of the right-handed ones for that neutrinos.<sup>10</sup> Of course, the reverse reasoning needs not be true.

Finally, we comment that the widely used mechanism for the smallness of the neutrino, see-saw mechanism [7], which needs two handedness of the neutrinos, may not exclude the possibility of the tachyonicity.

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<sup>10</sup> If we might find the single-handed spinorial tachyons without the right-handed partners, we must admit the necessity of the hypothetical number of Footnote 6 in order to describe consistently our nature, especially the tachyons. This situation may be compared to the case of the Schrödinger wave equation of the non-relativistic quantum theory, where the introduction of the pure-imaginary number is inevitable.

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